

Решение уравнения
по методу Фурье-перемещения уравнений.

Решение.

1. Решение уравнение:

$$a) \sqrt{2} \cos x - 1 = 0.$$

$$\cos x = \frac{1}{\sqrt{2}}.$$

$$x = \pm \arccos \frac{1}{\sqrt{2}} + 2\pi n, n \in \mathbb{Z}.$$

$$x = \pm \frac{\pi}{4} + 2\pi n, n \in \mathbb{Z}.$$

$$\text{Ответ: } \pm \frac{\pi}{4} + 2\pi n, n \in \mathbb{Z}.$$

$$b) 3 \operatorname{tg} 2x + \sqrt{3} = 0.$$

$$\operatorname{tg} 2x = -\frac{\sqrt{3}}{3}.$$

$$2x = \arctg\left(-\frac{\sqrt{3}}{3}\right) + k\pi, k \in \mathbb{Z}$$

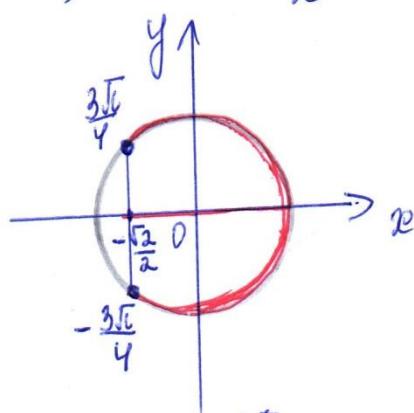
$$2x = -\frac{\pi}{6} + k\pi, k \in \mathbb{Z}$$

$$x = -\frac{\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

$$\text{Ответ: } -\frac{\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}.$$

2. Решение неравенства:

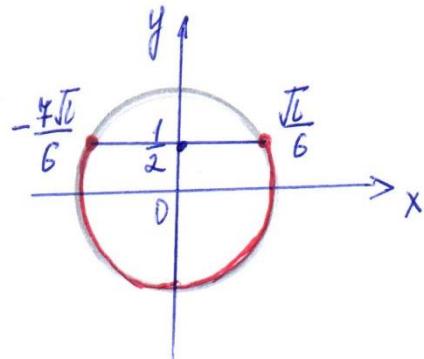
$$a) \cos x \geq -\frac{\sqrt{2}}{2}.$$



$$-\frac{3\pi}{4} + 2\pi n \leq x \leq \frac{3\pi}{4} + 2\pi n, n \in \mathbb{Z}.$$

$$\text{Ответ: } \left[-\frac{3\pi}{4} + 2\pi n; \frac{3\pi}{4} + 2\pi n\right], n \in \mathbb{Z}$$

$$b) \sin x \leq \frac{1}{2}.$$



$$-\frac{7\pi}{6} + 2\pi n \leq x \leq \frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}.$$

$$\text{Ответ:}$$

$$x \in \left[-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n\right],$$

$$n \in \mathbb{Z}.$$

3. Решение уравнений:

a) $\sin 2x - \sin x = 0$.

$$2\sin x \cos x - \sin x = 0.$$

$$\sin x (2\cos x - 1) = 0.$$

$$\begin{cases} \sin x = 0 \\ 2\cos x - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} \sin x = 0 \\ \cos x = \frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} x = \pi n \\ x = \pm \arccos \frac{1}{2} + \pi n \end{cases} \quad n \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} x = \pi n \\ x = \pm \frac{\pi}{3} + 2\pi n \end{cases} \quad n \in \mathbb{Z}.$$

Решение: $\pi n; \pm \frac{\pi}{3} + 2\pi n, \quad n \in \mathbb{Z}$.

b) $\sin^2 x - 3\sin x \cos x - 4\cos^2 x = 0 \mid : \cos^2 x$

$$\tan^2 x - 3\tan x - 4 = 0.$$

Пусть $\tan x = t$, тогда

$$t^2 - 3t - 4 = 0.$$

$$D = 9 + 16 = 25$$

$$t = \frac{3 \pm 5}{2} = 4; -1.$$

$$\begin{cases} \tan x = 4, \\ \tan x = -1; \end{cases} \Leftrightarrow \begin{cases} x = \arctan 4 + \pi n, \\ x = \arctan(-1) + \pi n, \end{cases} \quad n \in \mathbb{Z} \Leftrightarrow \begin{cases} x = \arctan 4 + \pi n, \\ x = -\frac{\pi}{4} + \pi n, \end{cases} \quad n \in \mathbb{Z}$$

Решение: $\arctan 4 + \pi n; -\frac{\pi}{4} + \pi n, \quad n \in \mathbb{Z}$.

$$8) 4 \cos x - 3 \sin x = 2 \quad | : \sqrt{4^2 + 3^2}$$

$$\frac{4}{\sqrt{25}} \cos x - \frac{3}{\sqrt{25}} \sin x = \frac{2}{\sqrt{25}}$$

ज्युक्स $\frac{4}{\sqrt{25}} = \sin \varphi$; $\frac{3}{\sqrt{25}} = \cos \varphi$

$$\varphi = \arcsin \frac{4}{\sqrt{25}} = \arccos \frac{3}{\sqrt{25}}$$

$$\sin \varphi \cos x - \cos \varphi \sin x = \frac{2}{\sqrt{25}}$$

$$\sin(\varphi - x) = \frac{2}{\sqrt{25}}$$

$$\varphi - x = (-1)^n \arcsin \frac{2}{\sqrt{25}} + \pi n, \quad n \in \mathbb{Z}$$

$$x - \varphi = (-1)^{n+1} \arcsin \frac{2}{\sqrt{25}} + \pi n, \quad n \in \mathbb{Z}$$

$$x = \varphi + (-1)^{n+1} \arcsin \frac{2}{\sqrt{25}} + \pi n, \quad n \in \mathbb{Z}$$

$$x = \arcsin \frac{4}{\sqrt{25}} + (-1)^{n+1} \arcsin \frac{2}{\sqrt{25}} + \pi n, \quad n \in \mathbb{Z}$$

परिवर्तन: $\arcsin \frac{4}{\sqrt{25}} + (-1)^{n+1} \arcsin \frac{2}{\sqrt{25}} + \pi n, \quad n \in \mathbb{Z}$

4. a) $6 \cos^2 x - 4 \cos x - 5 = 0$.

$$\cos x = t, \quad |t| \leq 1.$$

$$6t^2 - 4t - 5 = 0$$

$$\Delta = 49 + 120 = 169$$

$$t = \frac{4 \pm 13}{12} = -\frac{1}{2}; \frac{5}{3} - \text{не годожу.} \quad |t| \leq 1$$

$$\cos x = -\frac{1}{2}$$

$$x = \pm \arccos \left(-\frac{1}{2} \right) + 2\pi n, \quad n \in \mathbb{Z}$$

$$x = \pm \frac{2\pi}{3} + 2\pi n, \quad n \in \mathbb{Z}$$

δ) Определить корни, присущие множеству
множеству $[-\pi; 2\pi]$.

$$-\pi \leq \frac{2\pi}{3} + 2\pi n \leq 2\pi$$

$$-\pi \leq -\frac{2\pi}{3} + 2\pi n \leq 2\pi$$

$$-\frac{5\pi}{3} \leq 2\pi n \leq \frac{4\pi}{3} \quad | : \pi$$

$$-\frac{5}{3} \leq 2n \leq \frac{4}{3} \quad | : 2$$

$$-\frac{5}{6} \leq n \leq \frac{2}{3}$$

$$-\frac{1}{3} \leq 2n \leq \frac{8}{3} \quad | : 2$$

$$-\frac{5}{6} \leq n \leq \frac{4}{3}$$

$$-\frac{1}{6} \leq n \leq \frac{4}{3}$$

Т.к. $n \in \mathbb{Z}$, то $n=0$.

$n \in \mathbb{Z}$, то $n=0; 1$.

Если $n=0$, то $\frac{2\pi}{3} + 2\pi n = \frac{2\pi}{3} = x_1$.

Если $n=0$, то $x_2 = -\frac{2\pi}{3}$.

Если $n=1$, то $x_3 =$

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}.$$

Ответ: а) $\pm \frac{2\pi}{3} + 2\pi n$, $n \in \mathbb{Z}$.

$$\delta) -\frac{2\pi}{3}; \frac{2\pi}{3}; \frac{4\pi}{3}.$$

5. Решение уравнения:

$$\sin^4 x + \cos^4 x = \cos^2 2x + \frac{1}{4}.$$

$$\sin^4 x + \cos^4 x = (\cos^2 x - \sin^2 x)^2 + \frac{1}{4}$$

$$\sin^4 x + \cos^4 x = \cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x = f$$

$$2\sin^2 x \cos^2 x = \frac{1}{4} \quad | : 2$$

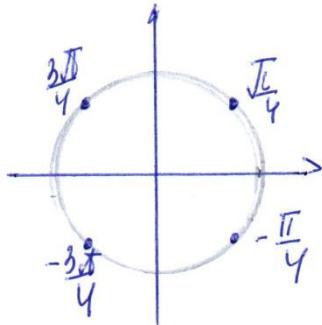
$$4\sin^2 x \cos^2 x = \frac{1}{2}$$

$$(2\sin x \cos x)^2 = \frac{1}{2}$$

$$\sin^2 2x = \frac{1}{2}.$$

$$\begin{cases} \sin 2x = \frac{1}{\sqrt{2}} \\ \cos 2x = -\frac{1}{\sqrt{2}} \end{cases} \quad \begin{cases} 2x = (-1)^n \arcsin \frac{1}{\sqrt{2}} + \pi n \\ 2x = (-1)^{n+1} \arcsin \frac{1}{\sqrt{2}} + \pi n \end{cases} \quad n \in \mathbb{Z}$$

$$\begin{cases} 2x = (-1)^n \frac{\pi}{4} + \pi n \\ 2x = (-1)^{n+1} \frac{3\pi}{4} + \pi n \end{cases} \quad n \in \mathbb{Z} \Rightarrow \begin{cases} 2x = \frac{\pi}{4} + \frac{\pi n}{2}, n \in \mathbb{Z} \\ x = \frac{\pi}{8} + \frac{\pi n}{4}, n \in \mathbb{Z} \end{cases}$$



Durchm: $\frac{\pi}{8} + \frac{\pi n}{4}, n \in \mathbb{Z}$